INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 39, Northern Spring 2018 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Six rooks are placed on a 6×6 board, so that none is under attack. Then, each unoccupied square is coloured red or blue according to the following rule:

> If all the rooks that attack that square are located at the same distance from the square, then it is coloured red. Otherwise it is coloured blue.

Is it possible that after colouring all the unoccupied squares, they are

(a) red? (1 point)

(b) blue? (2 points)

2. Let K be a point on the hypotenuse AB of a right-angled triangle ABC, and let L be a point on the side AC such that $AK = AC$ and $BK = LC$ respectively. Let M be the point of intersection of the line segments BL and CK. Prove that triangle CLM is isosceles. (4 points)

3. An integer has been written in each square of a 4×4 table. The sums of the numbers in each column and each row of the table are the same. Seven of the numbers in the table are known, while the rest have been lost (see diagram below).

1	?	?	2
?	4	5	?
?	6	7	?
3	?	?	?

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Is it possible having only the above information to restore

(a) at least one of the lost numbers? (2 points)

(b) at least two of the lost numbers? (2 points)

4. Three positive integers are given such that each of them is divisible by the greatest common divisor of the other two numbers, and the least common multiple of any two is divisible by the third number. Are these three numbers necessarily equal to each other? (4 points)

5. Thirty points have been chosen in the plane so that no three lie on the same line. Then 7 red lines are drawn so that they do not contain any of the chosen points. Is it possible that each line segment connecting two chosen points crosses at least one red line? (5 points)